

# Forecasting stock market prices: Lessons for forecasters \*

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**Abstract:** In recent years a variety of models which apparently forecast changes in stock market prices have been introduced. Some of these are summarised and interpreted. Nonlinear models are particularly discussed, with a switching regime, from forecastable to non-forecastable, the switch depending on volatility levels, relative earnings/price ratios, size of company, and calendar effects. There appear to be benefits from disaggregation and for searching for new causal variables. The possible lessons for forecasters are emphasised and the relevance for the Efficient Market Hypothesis is discussed.

**Keywords:** Forecastability, Stock returns, Non-linear models, Efficient markets.

## 1. Introduction: Random walk theory

For reasons that are probably obvious, stock market prices have been the most analysed economic data during the past forty years or so. The basic question most asked is – are (real) price changes forecastable? A negative reply leads to the random walk hypothesis for these prices, which currently would be stated as:

$H_{01}$ : Stock prices are a martingale.

$$\text{i.e. } E[P_{t+1} | I_t] = P_t,$$

where  $I_t$  is any information set which includes the prices  $P_{t-j}$ ,  $j \geq 0$ . In a sense this hypothesis has to be true. If it were not, and ignoring transaction costs then price changes would be consistently forecastable and so a money machine is created and indefinite wealth is possible. How-

ever, a deeper theory – known as the Efficient Market Hypothesis – suggests that mere forecastability is not enough. There are various forms of this hypothesis but the one I prefer is that given by Jensen (1978):

$H_{02}$ : A market is efficient with respect to information set  $I_t$  if it is impossible to make economic profits by trading on the basis of this information set.

By ‘economic profits’ is meant the risk-adjusted returns ‘net of all costs’. An obvious difficulty with this hypothesis is that it is unclear how to measure risk or to know what transaction costs are faced by investors, or if these quantities are the same for all investors. Any publically available method of consistently making positive profits is assumed to be in  $I_t$ .

This paper will concentrate on the martingale hypothesis, and thus will mainly consider the forecastability of price changes, or returns (defined as  $(P_t - P_{t-1} + D_t)/P_{t-1}$  where  $D_t$  is dividends), but at the end I will give some consideration to the efficient market theory. A good survey of this hypothesis is LeRoy (1989).

By the beginning of the seventies I think that it was generally accepted by forecasters and re-

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searchers in finance that the random walk hypothesis (or  $H_{01}$ ) was correct, or at least very difficult to refute. In a survey in 1972 I wrote, 'Almost without exception empirical studies...' support a model for  $p_t = \log P_t$  of the form

$$\Delta p_{t+1} = \theta \Delta p_t + 1_{t-1} + \epsilon_{t+1},$$

where  $\theta$  is near zero,  $1_t$  contributes only to the very low frequencies and  $\epsilon_t$  is zero mean white noise. A survey by Fama (1970) reached a similar conclusion. The information sets used were:

$I_{1t}$ : lagged prices or lags of logged prices.

$I_{2t}$ :  $I_{1t}$  plus a few sensible possible explanatory variables such as earnings and dividends.

The data periods were usually daily or monthly. Further, no profitable trading rules were found, or at least not reported. I suggested a possible reporting bias – if a method of forecasting was found an academic might prefer to profit from it rather than publish. In fact, by this period I thought that the only sure way of making money from the stock market was to write a book about it. I tried this with Granger and Morgenstern (1970), but this was not a financially successful strategy.

However, from the mid-seventies and particularly in the 1980s there has been a burst of new activity looking for forecastability, using new methods, data sets, longer series, different time periods and new explanatory variables. What is interesting is that apparent forecastability is often found. An important reference is Guimarães, Kingsman and Taylor (1989). The objective of this part is to survey some of this work and to suggest lessons for forecasters working on other series.

The notation used is:

$P_t$  = a stock price,

$p_t$  =  $\log P_t$ ,

$D_t$  = dividend for period  $t$ ,

$R_t$  = return =  $(P_t + D_t - P_{t-1})/P_{t-1}$ ,

[In some studies the return is calculated without the dividend term and approximated by the change in log prices.]

$r_t$  = return on a 'risk free' investment,

$R_t - r_t$  = excess return,

$\beta$  = risk level of the stock,

$R_t - r_t - \beta \times$  market return – cost of transaction = risk-adjusted profits.

The risk is usually measured from the capital asset pricing model (CAPM):

$$R_t - r_t = \beta (\text{market excess return}) + e_t,$$

where the market return is for some measure of the whole market, such as the Standard and Poor's 500.  $\beta$  is the non-diversifiable risk for the stock. This is a good, but not necessarily ideal, measure of risk and which can be time-varying although this is not often considered in the studies discussed below.

Section 2 reviews forecasting models which can be classified as 'regime-switching'. Section 3 looks at the advantages of disaggregation, Section 4 considers the search for causal variables, Section 5 looks at technical trading rules, Section 6 reviews cointegration and chaos, and Section 7 looks at higher moments. Section 8 concludes and re-considers the Efficient Market Theory.

## 2. Regime-switching models

If a stationary series  $x_t$  is generated by:

$$x_t = \alpha_1 + \gamma_1 x_{t-1} + \epsilon_t \quad \text{if } z_t \text{ in } A$$

and

$$x_t = \alpha_2 + \gamma_2 x_{t-1} + \epsilon_t \quad \text{if } z_t \text{ not in } A,$$

then  $x_t$  can be considered to be regime switching, with  $z_t$  being the indicator variable. If  $z_t$  is a lagged value of  $x_t$  one has the switching threshold autoregressive model (STAR) discussed in detail in Tong (1990), but  $z_t$  can be a separate variable, as is the case in the following examples. It is possible that the variance of the residual  $\epsilon_t$  also varies with regime. If  $x_t$  is a return (or an excess return) it is forecastable in at least one regime if either  $\gamma_1$  or  $\gamma_2$  is non-zero.

### 2.a. Forecastability with Low Volatility

LeBaron (1990) used  $R_t$ , the weekly returns of the Standard and Poor 500 index for the period 1946–1985, giving about 2,000 observations. He used as the indicator variable a measure of the recent volatility

$$\hat{\sigma}_t^2 = \sum_{i=0}^{10} R_{t-i}^2$$

and the regime of interest is the lowest one-fifth quantile of the observed  $\hat{\sigma}$  values in the first half of the sample. The regime switching model was estimated using the first half of the sample and post-sample true one-step forecasts were evaluated over the second half. For the low volatility regime he finds a 3.1 percent improvement in forecast mean squared error over a white noise with non-zero mean (that is, an improvement over a model in which price is taken to be a random walk with drift). No improvement was found for other volatility regimes. He first takes  $\alpha$  (the constant) in the model to be constant across regimes, relaxing this assumption did not result in improved forecasts. Essentially the model found is

$$R_t = \alpha + 0.18R_{t-1} + \epsilon_t \quad \text{if have low volatility}$$

$$R_t = \alpha + \epsilon_t \quad \text{otherwise,}$$

where  $\alpha$  is a constant. This non-linear model was initially found to fit equally well in and out of sample. However, more recent work by LeBaron did not find much forecasting ability for the model.

### 2.b. Earnings and size portfolios

Using the stocks of all companies quoted on either the New York or American Stock Exchanges for the period 1951 to 1986, Keim (1989) formed portfolios based on the market value of equity (size) and the ratio of earnings to price (E/P) and then calculated monthly returns (in percentages). Each March 31<sup>st</sup> all stocks were ranked on the total market value of the equity (price  $\times$  number of shares) and ten percent with the lowest ranks put into the first (or smallest) portfolio, the next 10% in the second portfolio and so forth up to the shares in the top 10% ranked giving the 'largest' portfolio. The portfolios were changed annually and average monthly returns calculated. Similarly, the portfolios were formed from the highest E/P values to the lowest (positive) values. [Shares of companies with negative earnings went into a separate portfolio.] The table shows the average monthly returns (mean) for five of the portfolios in each case, together

with the corresponding standard errors:

Size	E/P				
	Mean	(s.d.)	Mean	(s.d.)	
smallest	1.79	(0.32)	highest	1.59	(0.25)
2 <sup>nd</sup>	1.53	(0.28)	2 <sup>nd</sup>	1.59	(0.22)
5 <sup>th</sup>	1.25	(0.24)	5 <sup>th</sup>	1.17	(0.22)
9 <sup>th</sup>	1.03	(0.21)	9 <sup>th</sup>	1.11	(0.25)
largest	0.99	(0.20)	lowest	1.19	(0.28)
			negative earnings	(1.39)	(0.39)

Source: Keim (1989).

It is seen that the smallest (in size) portfolios have a substantially higher average return than the largest and similarly the highest E/P portfolios are better than the lowest.

The two effects were then combined to generate 25 portfolios, five were based on size and each of these was then sub-divided into five parts on E/P values. A few of the results are given in the following table as average monthly returns with beta risk values shown in brackets.

Size	E/P ratio		
	Lowest	Middle	Highest
smallest	1.62 <1.27>	1.52 <1.09>	1.90 <1.09>
middle	1.12 <1.28>	1.09 <1.02>	1.52 <1.06>
largest	0.89 <1.11>	0.97 <0.98>	1.43 <1.03>

Source: Keim (1989).

The portfolio with the highest E/P ratio and the smallest size has both a high average return and a beta value only slightly above that of a randomly selected portfolio (which should have a beta of 1.0). The result was found to hold for both non-January months and for January, although returns in January were much higher, as will be discussed in the next section. Somewhat similar results have been found for stocks on other, non-U.S. exchanges. It should be noted that as portfolios are changed each year, transaction costs will be moderately large.

The results are consistent with a regime-switching model with the regime determined by the size and E/P variables at the start of the year. However, as rankings are used, these variables for a single stock are related to the actual values of the variables for all other stocks.

### 2.c. *Seasonal effects*

A number of seasonal effects have been suggested but the strongest and most widely documented is the January effect. For example Keim (1989) found that the portfolio using highest E/P values and the smallest size gave an average return of 7.46 (standard error 1.41) over Januarys but only 1.39 (0.27) in other months. A second example is the observation that the small capitalization companies (bottom 20% of companies ranked by market value of equity) out-performed the S&P index by 5.5 percent in January for the years 1926 to 1986. These small firms earned inferior returns in only seven out of the 61 years. Other examples are given in Ikenberry and Lakonishok (1989). Beta coefficients are also generally high in January.

The evidence suggests that the mean of returns have regime changes with an indicator variable which takes a value of unity in January and zero in other months.

### 2.d. *Price reversals*

A number of studies have found that shares that do relatively poorly over one period are inclined to perform well over a subsequent period, thus giving price change reversals. A survey is provided by DeBondt (1989). For example, Dyl and Maxfield (1987) selected 200 trading days in random in the period January 1974 to January 1984, each day the three NYSE or AMEX stocks with the greatest percentage price loss (on average -12%) were noted. Over the next ten trading days, these losers earn a risk-adjusted return of 3.6 percent. Similarly the three highest gainers lost an average 1.8% over the next ten days. Other studies find similar evidence for daily, weekly and even monthly returns. Transaction costs will be fairly heavy and a strategy based on these results will probably be risky.

However, Lehman (1990) considered a portfolio whose weights depended on the return of a security the previous week minus the overall return, with positive weights on previous losers and negative weights (going short) on previous winners. The portfolio was found to consistently produce positive profits over the next week, with very few losing periods and so with small risk. Transaction costs were substantial but worthwhile prof-

its were achieved for transaction costs at a level appropriate for larger traders. Thus, after allowing for risk and costs, a portfolio based on price reversal was found to be clearly profitable.

Long term price reversals have also been documented. For example, Dark and Kato (1986) found in the Japanese market that for the years 1964 to 1980, the three year returns for decile portfolios of extreme previous losers exceed the comparable returns of extreme previous winners by an average 70 percent.

In this case the indicator variable is the extreme relative loss value of the share. As before the apparent forecastability leads to a simple investment strategy, but knowledge is required of the value taken by some variable based on all stocks in some market.

### 2.e. *Removal of extreme values*

It is well known that the stock markets occasionally experience extraordinary movements, as occurred in October 1987, for example. Friedman and Laibson (1989) point out that these large movements are of overpowering importance and may obscure simple patterns in the data. They consider the Standard and Poor 500 quarterly excess returns (over treasury bills) for the period 1954I to 1988IV. After removal of just four extreme values, chosen by using a Poisson model, the remaining data fits an AR(1) model with significant lag coefficient of 0.207 resulting in an  $R^2$  value of 0.036. The two regimes are thus the 'ordinary' excess returns, which seem to be forecastable, and the extra-ordinary returns which are not, from the lagged data at least.

## 3. **Benefits of disaggregation**

A great deal of the early work on stock market prices used aggregates, such as the Dow Jones or Standard and Poor indices, or portfolios of a random selection of stocks or some small group of individual stocks. The availability of fast computers with plenty of memory and tapes with daily data for all securities on the New York and American Exchanges, for example, allows examination of all the securities and this can on occasion be beneficial. The situation allows cross-section regressions with time-varying coefficients

which can possibly detect regularities that were not previously available. For example Jegadeesh (1990) uses monthly data to fit cross-section models of the form

$$R_{it} - \bar{R}_{it} = a_{0t} + \sum_{j=1}^{12} a_{jt} R_{i,t-j} + a_{13,t} R_{i,t-24} + a_{14,t} R_{i,t-36} + u_{it}$$

for each month. Thus, a lagged average relationship is considered with coefficients changing each month. Here  $\bar{R}_{it}$  is the average return over a long (four or six years) period which exclude the previous three years. [In the initial analysis,  $\bar{R}$  was estimated over the following few years, but this choice was dropped when forecasting properties were considered.] Many of the averaged  $a_j$  were significantly different from zero, particularly at lags one and twelve, but other average coefficients were also significant, including at lags 24 and 36. A few examples are shown, with  $t$ -values in brackets.

	$\bar{a}_1$	$\bar{a}_{12}$	$\bar{a}_{14}$	$R_c^2$
all months	-0.09(18)	0.034(9)	0.019(6.5)	0.108
January	-0.23 (9)	0.08 (5)	0.034(2.6)	0.178
Feb. to Dec.	-0.08(17)	0.03 (8)	0.017(6)	0.102

Source: Jegadeesh (1990).

There is apparently some average, time-varying structure in the data, as seen by  $R_c^2$  values of 10% or more. As noticed earlier, January has more forecastability than other months and it was found that a group of large firms had regressions with higher  $R_c^2$  in February to December than all firms using these regressions (without the  $\bar{R}$  terms), stocks were ranked each month on their expected forecastability and ten portfolios formed from the 10% most forecastable ( $P_1$ ), second 10% and so forth up to the 10% least forecastable ( $P_{10}$ ). The average abnormal monthly returns (i.e. after risk removal) on the 'best' and 'worst' portfolios for different periods were

	All months	January	Feb.-Dec.
$P_1$	0.011	0.024	0.009
$P_{10}$	-0.014	-0.020	-0.017

Source: Jegadeesh (1990).

There is thus seen to be a substantial benefit from using the best portfolio rather than the worst one based on the regressions. Benefits were also found, but less substantial ones, using twelve

month ahead forecasts. Once transaction costs are taken into account the potential abnormal returns from using  $P_1$  are halved, but are still around 0.45% per month (from personal communication by author of the original study).

#### 4. Searching for causal variables

Most of the studies discussed so far have considered forecasting of prices from just previous prices but it is also obviously sensible to search for other variables that provide some forecastability. The typical regression is

$$\Delta p_t = \text{constant} + \beta' X_{t-1} + \epsilon_t,$$

where  $X_t$  is a vector of plausible explanatory, or causal variables, with a variety of lags considered. For example Darrat (1990) considered a monthly price index from the Toronto Stock Exchange for the period January 1972 to February 1987 and achieved a relationship:

$$\begin{aligned} \Delta p_t = & 2.3 \Delta \text{volatility of interest rates } (t-1) \\ & (6) \\ & - 0.25 \Delta \text{ production index } (t-1) \\ & (5.3) \\ & + 0.35 \Delta \text{ long-term interest rate } (t-10) \\ & (4.2) \\ & - 0.015 \Delta \text{ cyclically-adjusted budget} \\ & (3.0) \\ & \text{deficit } (t-3), \end{aligned} \tag{4.1}$$

$$R^2 = 0.46,$$

$$\text{Durbin-Watson} = 2.01,$$

where only significant terms are shown and the modulus of  $t$ -values in brackets. Several other variables were considered but not found to be significant, including changes of short-term rates, inflation rate, base money and the US-Canadian exchange rate, all lagged once. An apparently high significance  $R^2$  value is obtained but no out-of-sample forecastability is investigated.

This search may be more successful if a long-run forecastability is attempted. For example Hodrick (1990) used monthly US data for the period 1929 to December 1987 to form NYSE value-weighted real market returns,  $R_{t+k,t}$  over the time span  $(t+1, t+k)$ . The regression

$$\log R_{t+k,t} = \alpha_k + \beta_k (\text{dividend/price ratio at } t)$$

found  $R^2$  increasing as  $k$  increases, up to  $R^2 = 0.354$  at  $k = 48$ . Thus, apparent long-run forecastability has been found from a very simple model. However, again no post-sample evaluation is attempted.

Pesaran and Timmerman (1990) also employ simple models that produce useful forecastability and they also conduct a careful evaluation of the model. As an example of the kind of model they produce, the following equation has as its dependent variable ( $Y_t$ ) the quarterly excess return on the Standard and Poor 500 portfolio:

$$\begin{aligned}
 Y_t = & -0.097 + 17.2 \text{ dividend yield } (t-2) \\
 & \quad (5.7) \\
 & - 1.59 \text{ inflation rate } (t-3) \\
 & \quad (2.8) \\
 & - 0.03 \text{ T-bill (end, } t-1) \\
 & \quad (6.2) \\
 & + 0.025 \text{ T-bill (begin, } t-2) \\
 & \quad (4.6) \\
 & + 0.066 \Delta \text{ twelve month bond state } (t-1) \\
 & \quad (5.5) \\
 & + \text{ residual,} \quad (4.2)
 \end{aligned}$$

$$R_c^2 = 0.364, \text{ Durbin-Watson} = 2.02.$$

Here dividend yield at time  $t$  is

$$\frac{\text{dividend on S\&P index } (t-1)}{\text{price of S\&P index } (t)}.$$

T-bill is the one month interest rate 'end' means it is measured at the end of the third month of the quarter, 'begin' indicates that it is measured at the end of the first month of the quarter. The two T-bill terms in the equation are thus effectively the change in the T-bill interest rate from one month to the next, plus one at the end of the quarter. As just lagged variables are involved and a reasonable  $R_c^2$  value is found, the model can potentially be used for forecasting. [It might be noted that  $R_c^2$  climbed to 0.6 or so for annual data.] Some experimentation with non-linear lagged dependent variables produced some increases in  $R_c^2$ , to about 0.39, but this more complicated model was not further evaluated.

A simple switching portfolio trading rule was considered:

- (i) Buy the S&P 500 index if the excess return was predicted to be positive according to equation (4.2), with the equation being sequentially re-estimated. Thus only data avail-

able at the time of the forecast was used in making the forecast.

- (ii) If the predictor was negative, the invest in T-bills.

The following table shows the rate of returns achieved by either using a 'buy-and-hold' market portfolio, or the switching portfolio obtained from the above trading rule or by just buying T-bills. As the switching rule involves occasional buying and selling, possibly quarterly, two levels of transaction costs are considered  $\frac{1}{2}\%$  and  $1\%$ .

	Investment strategy			T-bill
	Market	Switching		
Transaction costs	0	$\frac{1}{2}\%$	1%	0
Interest rate of returns	9.51	13.30	12.39	6.34
Standard deviation of returns	8.23	5.43	5.41	0.70
Wealth at end of period <sup>a</sup>	1394	3736	2961	595

<sup>a</sup> The period considered from 1960.I to 1988.IV and the wealth accumulates from an investment of \$100 in December 1959.

Source: Pesaran and Timmerman (1990).

Although the results presented are slightly biased against the switching portfolio zero transaction costs are assumed for the alternative investments, the trading rule based on the regression is seen to produce the greatest returns and as a lower risk-level than the market (S&P 500) portfolio. A variety of other evaluation methods and other regressions are also presented in the paper.

It would seem that dividend/price ratios and interest rates have quite good long-run forecasting abilities for stock price index returns.

## 5. A new look at old techniques – Technical trading rules

A strategy that is popular with actual speculators, but is disparaged by academics, is to use an automatic, or technical trading rule. An example is to use perceived patterns in the data, such as the famous 'head and shoulders', and to devise a rule based on them. Much technical analysis is difficult to evaluate, as the rules are not precise enough. The early literature did consider various simple rules but generally found little or no forecasting value in them. However, the availability of

fast computers has allowed a new, more intensive evaluation to occur, with rather different results.

Brock, Lakonishok and LeBaron (1991) consider two technical rules, one comparing the most recent value to a recent moving average, and the other is a 'trading range breakout'. Only the first of these is discussed here.

The first trading rule is as follows:

Let  $M_t$  = average of previous 50 prices, form a band  $B_t = (1 \pm 0.01) M_t$ , so that the band is plus and minus 1% around  $M_t$ . If  $P_t$ , the current price, is above the band, this is a buy signal, if it is below the band, this is a sell signal.

Using 90 years of daily data for the Dow–Jones Index (giving a sample of over twenty-three thousand values) for the period 1897 to 1986, the rule suggested buying 50% of the time giving an average return next day of 0.00062 ( $t = 3.7$ ) and selling 42% of the time, giving an average return of  $-0.00032$  ( $t = 3.6$ ). The return on the rule 'buy if have buy signal and go short on a sell signal' gave an average daily return of 0.00094 ( $t = 5.4$ ). The first two  $t$ -values are for the return minus the daily unconditional average return, the 'buy-sell'  $t$ -value is relative to zero. If this buy-sell strategy was used 200 times a year, it gives a return of 20.7 percent for the year. However, this figure ignores transaction costs, which could be substantial. The trading rule was considered for four sub-periods and performed similarly for the first three but less well for the most recent sub-period of 1962–1986, where the buy-sell strategy produced a daily return of 0.00049. Other similar trading rules were considered and gave comparable results. Thus, this rule did beat a buy-and-hold strategy by a significant amount if transaction costs are not considered. The authors also consider a much more conservative rule, with a fixed ten day holding period after a buy or sell signal. The above rule then averages only  $3\frac{1}{2}$  buy and sell signals a year, giving an annual expected return of 8.5% compared to an annual return for the Dow Index of about 5%, again ignoring transaction costs. These, and the results for the other trading rules considered suggest that there may be regular but subtle patterns in stock price data, which would give useful forecastability. However, very long series are needed to investigate these rules.

Neftci (1991) investigates a similar moving average trading rule using different statistical methods and an even longer period – monthly Dow–Jones Industrial Index starting in 1792, up to 1976. Let  $M_t$  be an equi-weighted moving average over the past five months. If  $P_t$  is the price of the index in month  $t$ , define a dummy variable:

$$\begin{aligned} D_t &= 1 \quad \text{if } P_t > M_t \text{ given } P_{t-1} < M_{t-1} \\ &= -1 \quad \text{if } P_t < M_t \text{ given } P_{t-1} > M_{t-1} \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Regression results are presented for the equation

$$P_{t+18} = \sum_{j=0}^{14} \alpha_j P_{t-j} + \sum_{j=0}^6 \gamma_j D_{t-j} + \text{residual,}$$

where the residual is allowed to be a moving average of order 17, for each of the three sub-periods 1792–1851, 1852–1910 and 1910–1976. In each case the sum of the alphas is near one, as suggested by the efficient market theory and in the more recent period the gammas were all significant, individually and jointly, suggesting some nonlinearity in the prices. No forecasting exercise was considered using the models. The use of data with such early dates as 1897 or 1792 is surely only of intellectual interest, because of the dramatic institutional changes there have occurred since then.

Neftci also proves, using the theory of optimal forecasts, that technical trading rules can only be helpful with forecasting if the price series are inherently nonlinear.

## 6. New techniques – Cointegration and chaos

Since the early statistical work on stock prices, up to 1975, say, a number of new and potentially important statistical models and techniques have been developed. Some arrive with a great flourish and then vanish, such as catastrophe theory, whereas others seem to have longer staying power. I will here briefly consider two fairly new approaches which have not been successful, so far, in predicting stock prices.

An  $I(1)$  series is one such that its first difference is stationary. A pair,  $X_t, Y_t$ , of  $I(1)$  series are called cointegrated if there is a linear combination of them,  $Z_t = X_t - AY_t$ , say, which is  $I(0)$ .

The properties and implications of such series are described in Granger (1986), Engle and Granger (1991), and many other publications in econometrics, macroeconomics and finance. If the series are cointegrated there is necessarily an error-correction data generating model of the form (ignoring constants):

$$\Delta X_t = \alpha_x Z_{t-1} + \text{lagged } \Delta X_t, \Delta Y_t \text{ terms} \\ + \text{white noise,}$$

plus a similar equation for  $\Delta Y_t$ , with at least one of  $\alpha_x, \alpha_y$  being non-zero. It follows that either  $X_t$  must help forecast  $Y_{t+1}$  or  $Y_t$  must help forecast  $X_{t+1}$  or both. Thus, if dividends and stock prices are found to be cointegrated, as theory suggests, then prices might help forecast dividends, which would not be surprising, but dividends not help forecast prices, in agreement with the efficient market hypothesis. However, for the same reason one would not expect a pair of stock prices to be cointegrated, as this would contradict the efficient market hypothesis. In fact several papers have been produced that claim to find cointegration between pairs of prices or of portfolios, but the error-correction models are not presented or the forecasting possibility explored, and so this work will not be surveyed. It should be noted that cointegration would be inconsistent with the well-respected capital asset pricing model (CAPM) which says that the price  $P_{it}$  of the  $i^{\text{th}}$  asset is related to the price of the whole market  $P_{mt}$  by

$$\Delta \log P_{it} = b_i \Delta \log P_{mt} + e_{it},$$

where  $e_{it}$  is white noise. Summing over time gives

$$\log P_{it} = b_i \log P_{mt} + \sum_{j=0}^t e_{i,t-j}.$$

As the last term is the accumulation of a stationary series, it is  $I(1)$  (ignoring trends) and so cointegration should not occur between  $\log P_{it}$  and  $\log P_{mt}$ . Similarly, there should be no cointegration between portfolios. Gonzalo (1991) has found no cointegration between three well known aggregates, the Dow-Jones Index, the Standard and Poor 500 Index and the New York Stock Exchange Equal Value Index.

A class of processes generated by particular deterministic maps, such as

$$y_t = 4y_{t-1}(1 - y_{t-1})$$

with

$$0 < y_0 < 1,$$

have developed a great deal of interest and can be called 'white chaos'. These series have the physical appearance of a stochastic independent (i.i.d.) process and also the linear properties of a white noise such as zero autocorrelations and a flat spectrum. The question naturally arises of whether the series we have been viewing as stochastic white noise are actually white chaos and are thus actually perfectly forecastable – at least in the short run and provided the actual generating mechanism is known exactly. The literature on chaos is now immense, involves exciting and deep mathematics and truly beautiful diagrams, and also is generally optimistic, suggesting that these processes occur frequently. In fact, a clear case can be made that they do not occur in the real world, as opposed to in laboratory physics experiments. There is no statistical test that has chaos as the null hypothesis. There also appears to be no characterizing property of a chaotic process, that is a property that is true for chaos but not for any completely stochastic process. These arguments are discussed in Liu, Granger and Heller (1991). It is true that some high-dimensional white chaotic processes are indistinguishable from iid series, but this does not mean that chaos occurs in practice. In the above paper, a number of estimates of a statistic known as the correlation dimension are made for various parameter values using over three thousand Standard and Poor 500 daily returns. The resulting values are consistent with stochastic white noise (or high dimensional chaos) rather than low dimensional – and thus potentially forecastable – white chaos. A little introspection also make it seem unlikely to most economists that a stock market, which is complex, involving many thousands of speculations, could obey a simple deterministic model.

## 7. Higher moments

To make a profit, it is necessary to be able to forecast the mean of price changes, and the studies reviewed above all attempt to do this. The efficient market theory says little about the fore-



castability of functions of price changes or returns, such as higher moments. If  $R_t$  is a return it has been found that  $R_t^2$  and  $|R_t|$  are clearly forecastable and  $|R_t|$  even more so, from lagged values. Taylor (1986) finds evidence for this using U.S. share prices and Kariya, Tsukuda and Maru (1990) get similar results for Japanese stocks. For example, if  $R_t$  is the daily return from the U.S. Standard and Poor index the autocorrelations for  $R_t$  are generally very small, the autocorrelations for  $R_t^2$  are consistently above 0.1 up to lag 100 and for  $|R_t|$  are about 0.35 up to lag 100. It is clear that these functions of returns are very forecastable, but this is not easily converted into profits, although there are implications for the efficiency of options markets. The results are consistent with certain integrated GARCH models but this work is still being conducted and the final conclusions have yet to be reached.

## 8. Lessons for forecasters

Despite stock returns once having been thought to be unforecastable, there is now plenty of optimism that this is not so, as the examples given above show. Is this optimism justified, and if yes, what are the lessons for forecasters working with other data sets? As there is an obvious possible profit motive driving research into the forecastability of stock prices, or at least returns, one can expect more intensive analysis here than elsewhere. Whereas too many forecasters seem to be content with just using easily available data, with the univariate or simple transfer function forecasting techniques that are found on popular computer packages, stock market research is more ambitious and wide-ranging. It should be emphasized that the above is not a complete survey of all of the available literature.

The sections above suggest that benefits can arise from taking a longer horizon, from using disaggregated data, from carefully removing outliers or exceptional events, and especially from considering non-linear models. Many of the latter can be classified as belonging to a regime switching model of the form

$$\text{Return}_t = \beta'_1 \underline{X}_{t-1} + (\beta'_2 \underline{X}_{t-1}) \Phi(w_{t-d}),$$

where  $\Phi(w)$  is a smooth monotonic function such

as a cumulative distribution function of a continuous random variable so that  $0 \leq \Phi \leq 1$ ,  $\underline{X}_t$  is a vector of explanatory variables possibly including lagged returns and  $w_t$  is the 'switching variable', possibly a lagged component of  $\underline{X}_t$  or some linear combination of these components. It has always been important to discover appropriate explanatory variables  $\underline{X}_t$  and with this new class of models it is especially important to find the appropriate switching variable, if it exists and is observable. This class of models is discussed in Granger and Teräsvirta (1992), where tests and estimation procedures are outlined.

The papers also suggest that some sub-periods may be more forecastable than others – such as summer months or January – and this is worth exploring. If many component series are available, then ranks may produce further information that is helpful with forecasting. There seems to be many opportunities for forecasters, many of whom need to break away from simple linear univariate ARIMA or multivariate transfer functions. It is often not easy to beat convincingly these simple methods, so they make excellent base-line models, but they often can be beaten.

Before the results discussed in previous sections are accepted the question of how they should be evaluated has to be considered. Many of the studies in this, and other forecasting areas, are of the 'if only I had known this at the beginning of the period I could have made some money' classification. For a 'forecasting model' to be accepted it has to show that it actually forecasts, it is not sufficient to produce a regression model evaluated only in sample. There is always the possibility of small-sample in-sample biases of coefficients which give overly encouraging results, as shown by Nelson and Kim (1990). The possibility of 'data mining' having occurred, with many models having been considered, and just the best one presented is also a worry. Only out-of-sample evaluation is relevant and, to some extent, avoids these difficulties. It is surprising that more of the studies surveyed do not provide results of forecasting exercises.

Not only do the models that are proposed as providing useful forecasts of price changes or returns need to be evaluated, to provide profitable strategies the forecast returns need to be corrected for risk levels and also for transaction costs. Many of the studies discussed earlier fail to

do this, and so, at present, say nothing about the correctness of the efficient market hypothesis (EMH). However, this criticism does not always apply, for example for the carefully conducted analysis by Pesaran and Timmerman (1990). Does this mean that the EMH should be rejected? One has to say – not necessarily, yet. If a method exists that consistently produces positive profits after allowing for risk correction and transaction costs and if this method has been publicly announced for some time, then this would possibly be evidence against EMH. There are so many possibly relevant trading rules that it is unrealistic to suppose that investors have tried them all, especially those that have only been discovered by expensive computation and sophisticated statistical techniques. Once knowledge of an apparently trading rule becomes wide enough, one would expect behaviour of speculators to remove its profitability, unless there exists another trading rule the speculators think is superior and thus concentrate on it. Only if a profitable rule is found to be widely known and remains profitable for an extended period can the efficient market hypothesis be rejected. It will be worthwhile checking in a few years on the continued profitability of the rules discussed earlier. This research program agrees with the modern taste in the philosophy of science to try to falsify theories rather than to try to verify them. Clearly verification of EMH is impossible.

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